# Strong (Weak) Domination in Soft Graph

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**Abstract**

In this paper, the concepts of strong (weak) domination in soft graph and some types of strong (weak) domination in soft graphs including independent strong (weak) dominating set, connected strong (weak) dominating set, triple connected strong (weak) dominating set in soft graphs are introduced and investigating some of their properties and proposed algorithm for strong minimal dominating set in soft graph.

***Keywords*—** Strong (weak) domination in soft graph, independent strong (weak) dominating set, connected strong (weak) dominating set, triple connected strong (weak) dominating set in soft graphs.

## I. Introduction

Akram and Nawaz introduced the novel concepts called fuzzy soft graphs and fuzzy vertex induced soft graphs [1] and further more Maji, Biswas and Roy[4] worked on soft set theory. A new notion on soft graph using soft sets was introduced by Rajesh K. Thumbakara and Bobin George[10]. Domination is an area in graph theory with an extensive research activity.The concept of domination in graphs is introduced by Haynes et al. (1998). In 2012, Bounds on connected domination in square of a graph is introduced by M.H.Muddabihal and G.Srinivasa. Triple connected domination number of a graph introduced by G.Mahadevan, Selvam [6]. The concept of strong and weak domination was introduced by E. Sampathkumar and L. Pushpa Latha [10]. The concept of domination in soft graphs is introduced by Sarala.N, Manju.K [15]. In this paper, we introduce the concept of strong (weak) domination in soft graphs and describe some types of domination in soft graph with related results and proposed algorithm to find strong minimal dominating set in soft graph.

## II. Preliminaries

### *Definition: 2.1*

Let U be a nonempty finite set of objects called Universe and let E be a nonempty set called parameters. An ordered pair (F, A) is said to be a Soft set over U, where F is a mapping from E into the set of all subsets of the set U. That is F: A → $\rho(U)$. Where $\rho(U)$ denotes the collection of all subsets of U. The set of all Soft sets over U is denoted by S (U).

### *Definition: 2.2*

Let $G = (V, E)$ be a graph and (F, A) be a soft set over V. Then (F, A) is said to be a soft graph of G if the subgraph induced by F(x) in G is a connected subgraph of G for all x ∈ A.

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***Definition: 2.3***

A set D of vertices of a soft graph (F, A) is said to be a dominating set if every vertex of the subgraph induced by F(x) in V −D is adjacent to a vertex in D.

***Example: 2.1***

Consider a graph G (V, E) shown in the figure 2.1

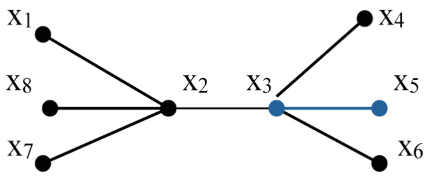



**Fig 2.1** simple tree graph

Let A = $\{v_1, v_2\} \subseteq V$ and (F,A) be a soft set over V with approximate function F : A → P(V ) by F $(v_1) = \{ x_8, x_2, x_3, x_5\}$, $F(v_2) = \{ x_1, x_2, x_3, x_4, x_6, x_7\}$


F $(v_1)$, Corresponding to parameter $v_1$. F $(v_2)$, corresponding to parameter $v_2$

**Fig 2.2** parameterised graph

Here $\{x_2, x_3\}$ is the dominating set of parameterised graph $F(v_1)$ and $F(v_2)$ . Therefore $\{x_2, x_3\}$ is the dominating set of soft graph (F, A)

***Definition: 2.4***

A dominating set D in a soft graph (F, A) is said to be a minimal dominating set if no proper subset of D is a dominating set.

***Definition: 2.5***

A dominating set $D \subseteq V$ is an independent set of a soft graph (F, A), if $\forall$ u, v $\in$ D, $N(u) \cap \{v\} = \varphi$. A dominating set which is independent is called an independent dominating set. The minimum cardinality of an independent dominating set in (F, A) is called the independent domination number i(F, A) of soft graph (F,A).

***Definition : 2.6***

A dominating set D of a soft graph (F, A) is said to be an connected dominating set of (F, A) If in each induced subgaraph of F(x) the induced subset of D is also connected.

***Definition: 2.7***

A set D⊆V is a weak dominating set, if every vertex u not in D is adjacent to a vertex v in D where deg(v)≤deg(u). A set D⊆V is a strong dominating set, if every vertex u not in D is adjacent to a vertex v in D where deg(v)≥deg(u).

***Definition: 2.8***

A subset D of V of a nontrivial connected graph G is said to be triple connected dominating set, if D is a dominating set and the induced sub graph <D> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by tc(G).

III. **Strong (weak) domination in soft graph**

***Definition:3.1***

Let (F, A) be a soft graph of G and u, v ∈ V. Then, u strongly dominates v and v weakly dominates u if (i) uv ∈ E and (ii) deg(u)≥deg(v). A set D of vertices of a soft graph (F, A) is said to be a strong (Weak) dominating set if every vertex of the subgraph induced by F(x) in V −D strongly (Weakly) dominated by at least one vertex in D. The strong (weak) domination number 7s (7w) of a soft graph (F, A) is the minimum cardinality of a strong (Weak) dominating set.

***Example: 3.1***

Consider a graph G (V, E) shown in the figure 3.1

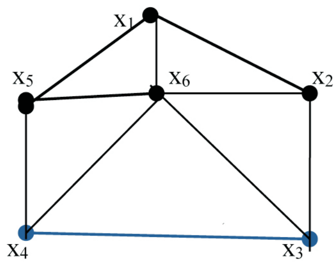
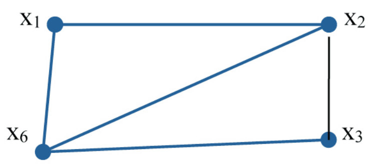
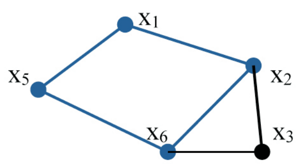



**Fig 3.1** simple graph

Let A = {$v_1$, $v_2$} ⊆ V and (F,A) be a soft set over V with approximate function F : A → P(V ) by F(x) = {y ∈ V : x adjacent to y} for all x ∈ A. That is, F ($v_1$) = { $x_1$, $x_2$, $x_3$, $x_6$}, F ($v_2$) = { $x_2$, $x_3$, $x_5$, $x_1$,$x_6$}




F ($v_1$), Corresponding to parameter $v_1$.




F ($v_2$), corresponding to parameter $v_2$

**Fig 3.2** parameterised graph

Here $\{x_6, x_2\}$ is the strong dominating set of parameterised graph F ($v_1$) and F($v_2$). Therefore $\{x_6, x_2\}$ the dominating sets of soft graph (F, A). Also $\{x_1, x_3\}$ is the weak dominating set of parameterised graph F ($v_1$) and F($v_2$). Therefore $\{x_1, x_3\}$ is the weak dominating set of soft graph (F, A).

***Definition: 3.2***

A strong (Weak) dominating set D in a soft graph (F, A) is said to be a minimal strong (Weak) dominating set if no proper subset of D is a strong (Weak) dominating set.

***Example: 3.2***

Consider a graph G (V, E) shown in the figure 3.3

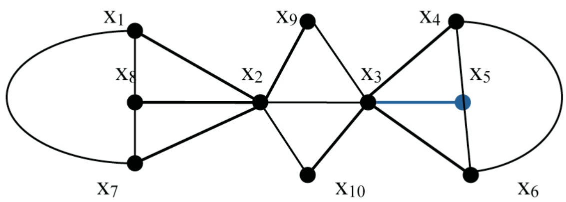



**Fig 3.3** simple tree graph

Let A = { $v_1$, $v_2$} ⊆ V and (F,A) be a soft set over V with approximate function F : A → P(V ) by F ($v_1$) = { $x_8$, $x_1$, $x_2$, $x_3$, $x_5$, $x_9$, $x_{10}$}, F($v_2$) = { $x_1$, $x_2$, $x_3$, $x_4$, x, $x_7$, $x_9$, $x_{10}$}


F ($v_1$), Corresponding to parameter $v_1$, F ($v_2$), corresponding to parameter $v_2$

**Fig 3.4** parameterised graph

Here $\{x_2, x_3\}$ is the minimal strong dominating set of parameterised graph F($v_1$) and F($v_2$). Therefore $\{x_2, x_3\}$ is the minimal strong dominating set of soft graph (F, A) and $\{x_1, x_5. x_9, x_{10}\}$ is the minimal weak dominating set of parameterised graph F($v_1$) and F($v_2$). Therefore $\{x_1, x_5. x_9, x_{10}\}$is the minimal weak dominating set of soft graph (F, A).

***Definition: 3.3***

The minimum cardinality of a strong (Weak) dominating set of a soft graph (F, A) is called the strong (Weak) domination number of (F,A) and is denoted by 7s (F, A) or 7w (F, A).

***Example: 3.3***

In example 3.2 the minimal strong dominating set is$\{x_2, x_3\}$.Therefore the strong domination number is 7s (F, A) = 2 and the minimal weak dominating set is $\{x_1, x_5. x_9, x_{10}\}$.Therefore the weak domination number is 7w (F, A) = 4

***Theorem: 3.1***

If D is a strong dominating set of a soft graph (F, A) and v ∈ V is the only vertex of maximum degree in every induced subgraph of F(x), then v ∈ D.

***Proof:***

Let v be the vertex of maximum degree in every induced subgraph of F(x) and D be a strong dominating set.To prove: v ∈ D. Suppose v ∉ D implies that v ∈ V −D. As v is the only vertex with maximum degree it will be strongly dominated by itself only. Therefore, if v ∉ D then there is no vertex in D which strongly dominates v. That is, D is not an strongly dominating set which contradicts to our assumption that D is a strong dominating set. Hence v ∈ D.

***Theorem: 3.2***

Let v be a vertex with deg (v) = Δ (F, A) = k and v is not adjacent to any other vertex of degree k then v must be in strongly dominating set in (F,A).

***Proof:***

Let v be any vertex of maximum degree k in (F, A) which is not adjacent to any vertex of the same degree k. That is, deg(v) ≥ deg(w), ∀w ∈ N(v). So, the vertex v is strongly dominated by itself only. Hence v must be in an strongly dominating set.

***Theorem: 3.3***

Let (F, A) be a soft graph of order n such that Δ(F,A) = k and there are r mutually non adjacent vertices with degree k such that there is no vertex which is strongly dominated by any two or more vertices of degree k then $r \leq \gamma_{sd}$ (F, A) ≤ n−rΔ(F,A).

***Proof:***

By Theorem 3.2 all the mutually nonadjacent vertices of degree k must be in every strongly dominating set. Therefore, $r \leq \gamma_{sd}$(F, A). Let v be any vertex of maximum degree k in every induced subgraph of F(x) which is not adjacent to any vertex of the same degree k and there is no vertex which is strongly dominated by any two or more vertices of degree k. Therefore, each vertex of degree k strongly dominates k + 1 distinct vertices from V. If we consider r vertices of maximum degree in an strongly dominating set then r + rΔ(F, A) vertices are strongly dominated. If r + rΔ(F, A) ≤ n then n−(r + rΔ(F, A)) number of vertices are not strongly dominated. To strongly dominate all the vertices of V we need to consider at least r + n−(r + rΔ(F, A)) vertices from V. Hence, $\gamma_{sd}$(F, A) ≤ n−rΔ(F, A).

***Definition: 3.4***

A strong (Weak) dominating set D ⊆ V is an independent set of a soft graph (F,A), if ∀u,v ∈ D, N(u)∩{v} = φ. A dominating set which is independent is called an strong (Weak) independent dominating set. The minimum cardinality of an independent strong (Weak) dominating set in (F, A) is called the strong (Weak) independent domination number $i_{sd}$(F, A) or $i_{wd}$(F, A) soft graph (F,A).

***Example: 3.4***

Consider a graph G (V, E) shown in the figure 3.5

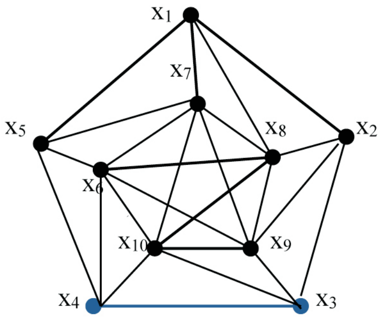



**Fig 3.5** simple graph

Let A = {$v_1$, $v_2$} ⊆ V and (F,A) be a soft set over V with approximate function F : A → P(V ) by F(x) = {y ∈ V : x adjacent to y} for all x ∈ A. That is, F ($v_1$) = { $x_2$,$x_4$, $x_5$, $x_6$, $x_7$, $x_8$, $x_9$, $x_{10}$}, F ($v_2$) = { $x_2$,$x_4$, $x_5$, $x_6$, $x_7$, $x_8$, $x_9$, $x_{10}$}


F ($v_1$), Corresponding to parameter $v_1$, F ($v_2$), corresponding to parameter $v_2$

**Fig 3.6** parameterised graph

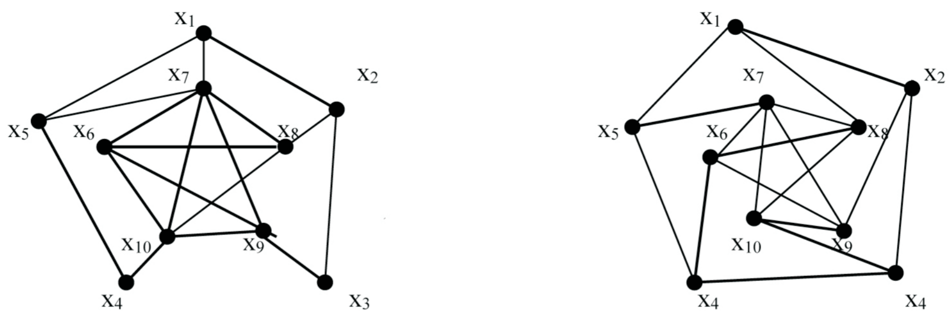
Here {$x_6$, $x_{10}$} is the strong independent dominating set of parameterised graph F ($v_1$) and F($v_2$). Therefore {$x_6$, $x_{10}$} the strong independent dominating set of soft graph (F, A). Also {$x_4$, $x_5$, $x_8$, $x_9$} is the weak independent dominating set of parameterised graph F ($v_1$) and F($v_2$). Therefore {$x_4$, $x_5$, $x_8$, $x_9$} is the weak independent dominating set of soft graph (F,A).

***Definition: 3.5***

A dominating set D of a soft graph (F, A) is said to be a **strong (Weak) connected dominating set** of (F, A) If in each induced subgaraph of F(x) the induced subset of D is also connected.

***Example: 3.5***

From figure 3.5, let A = {$v_1$, $v_2$} ⊆ V and (F,A) be a soft set over V with approximate function F : A → P(V ) by F ($v_1$) = { $x_1$, $x_2$, $x_3$, $x_4$,$x_5$, $x_6$, $x_7$, $x_8$, $x_9$,$x_{10}$}, F($v_2$) = { $x_1$, $x_2$, $x_3$, $x_4$,$x_5$, $x_6$, $x_7$, $x_8$, $x_9$,$x_{10}$}




F ($v_1$), Corresponding to parameter $v_1$ F ($v_2$), corresponding to parameter $v_2$

**Fig 3.7** parameterised graph

Here {$x_6$, $x_7$, $x_8$, $x_9$, $x_{10}$} is the strong connected dominating set of parameterised graph F ($v_1$) and F($v_2$) . Therefore {$x_6$, $x_7$, $x_8$, $x_9$, $x_{10}$} the strong connected dominating sets of soft graph (F, A). Also { $x_1$, $x_2$, $x_3$, $x_4$,$x_5$} is the weak connected dominating set of parameterised graph F ($v_1$) and F($v_2$) . Therefore { $x_1$, $x_2$, $x_3$, $x_4$,$x_5$}is the weak connected dominating set of soft graph (F, A)

***Theorem: 3.4***

Let D be a minimal strong dominating- set of a soft graph (F, A). Then for each u∈D of the following holds.

(i) No vertex in D strongly dominate v
(ii) There exists v∈ V-D such that v is the only vertex in D which strongly dominates u.

***Proof:***

Suppose D is a minimal connected dominating set of a soft graph (F, A). Then for each node u∈D the set D′=D-{u} is not a connected dominating set. Thus, there is a node v∈V-D′ which is not dominated by any node in D′. Now either u=v or v∈V-D, If v=u then no vertex in D strongly dominates v. If v∈V-D and v is not dominated by D-{u} but is dominated by D, Then u is the only strong neighbour of v and v is the only vertex in D which strongly dominates u.

Conversely suppose D is a dominating set and each node u∈D, one of the two stated conditions holds. Now we prove D is a minimal strong connected dominating set. Suppose D is not a minimal strong connected dominating set, then there exists a node u∈D such that D-{u} is a dominating set. Therefore condition (i) does not hold. Also if D-{u} is a dominating set then every node in V-D is a strong neighbour to at least one node in D-{u}. Therefore condition (ii) does not hold. Hence neither condition (i) nor (ii) holds which is a contradiction.

***Definition:3.6***

A dominating set D of a soft graph (F, A) is said to be **triple connected dominating set**. If D is the dominating set and the induced sub graph <D> is triple connected .The minimum cardinality taken over all triple connected dominating set of (F, A) is called the triple connected dominating number of (F, A) and is denoted by $\square_{tc}$(F, A).

***Example: 3.6***

In example 3.5 $\{x_6, x_7\}$ is the triple connected dominating set of parameterised graph $F(v_1)$ and $F(v_2)$. Therefore $\{x_6, x_7\}$ the triple connected dominating set of soft graph (F, A).

***Definition: 3.7***

A dominating set D of a soft graph (F, A) is said to be an **strong (Weak) triple connected dominating set**, if D is a strong (weak) dominating set and the induced sub graph <D> is a triple connected. The minimum cardinality taken over all strong (weak) triple connected dominating set is called the strong (weak) triple connected domination number and it denoted by $\gamma_{stc}(F, A)$ or $\gamma_{wtc}(F, A)$.

***Example: 3.7***

In example 3.5 $\{x_6, x_7\}$ is the strong triple connected dominating set of parameterised graph F $(v_1)$ and $F(v_2)$ . Therefore $\{x_6, x_7\}$ the strong triple connected dominating set of soft graph (F, A). Also $\{x_4,x_5\}$ is the weak triple connected dominating set of parameterised graph F $(v_1)$ and $F(v_2)$ . Therefore $\{x_4,x_5\}$is the weak triple connected dominating set of soft graph (F, A)

***Theorem 3.5***

Let (F, A) be an soft graph and each induced subgraph of F(x) with P vertices and strong (weak) triple connected dominating set D with vertices P if any only if (F, A) is isomorphic to the soft graph with a path on 3 vertices or a cycle on 3 vertices.

***Proof:***

Suppose (F, A) is isomorphic to the soft graph with a path on 3 vertices or a cycle on 3 vertices, then the strong(weak) triple connected dominating vertices is 3, and P vertices. Conversely, Let (F, A) be an soft graph and each induced subgraph of F(x) with P vertices such that strong (weak) triple connected dominating vertices P then (F, A) $\cong$ to the soft graph with a path on 3 vertices or a cycle on 3 vertices.

***Theorem 3.6***

For any soft graph (F, A) with $P\geq 3$ vertices and exactly one vertex has $\Delta_N(F, A) \leq P-2$, the strong triple connected dominating vertices is 3.

***Proof:***

Let (F, A) be an soft graph with parameterised graphs F(a) and F(b) . Let F(a) with $P\geq 3$ vertices and exactly one vertex has maximum neighborhood degree $\Delta_N(F(a)) \leq P-2$. Let v be the vertex of maximum neighborhood degree $\Delta_N(F(a)) \leq P-2$. Let $v_1,v_2$...... and $v_{p-2}$ be the vertices which are adjacent to v and Let $v_{p-1}$ be the vertex which is not adjacent to v. Since F(a) is connected, $v_{p-1}$ is adjacent to a vertex $v_i$ for some i, Then $s = \{v,v_i,v_{p-1}\}$ forms a minimum strong triple connected dominating set of F(a).Same is true for F(b). Hence $s = \{v,v_i,v_{p-1}\}$ forms a minimum strong triple connected dominating set of (F. A)

## IV. Algorithm to find strong minimal dominating set in soft graph

Let (F, A) be a soft graph and D be a dominating set of all induced subgraphs of F(x)

***Step: 1*** First we have to initialize a set D as empty set

***Step: 2*** Take any maximum degree vertex $v_i$ strongly dominates $u_i$ which lies in all induced subgraphs of F(x)

such that $deg(v_i) \geq deg(u_i)$ and delete all the edges adjacent to $v_i$

***Step: 3*** Take $v_i$ is the one of the vertex in dominating set D.

***Step: 4*** Repeat the step 2 to 3 untill all the vertex sets of induced subgraphs of F(x) become empty.

***Step: 5*** Now all the vertices in the resulting set D in the soft graph (F, A) form a minimal strong dominating set.

***Example: 4.1***

Consider a graph G (V, E) shown in the figure 3.1

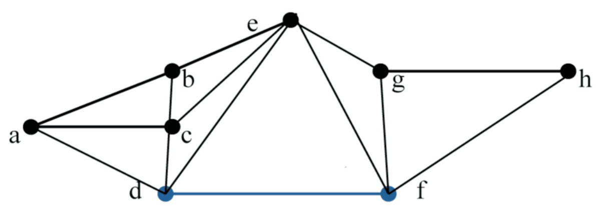



**Fig 4.1** simple graph

Let $A = \{ v_1, v_2\} \subseteq V$ and (F,A) be a soft set over V with approximate function $F : A \rightarrow P(V)$ by

$F(x) = \{y \in V : xRy / x \text{ adjacent to } y\}$ for all $x \in A$. That is, F(a) = { a,b,c,d,e,f}, F(b) = {b,c,d,e,f,g}


F ($v_1$), Corresponding to parameter $v_1$

F($v_2$), corresponding to parameter $v_2$

**Fig 4.2** parameterised graph

***Step:1*** First we have to initialize a set D as empty set

***Step:2*** Take vertex e with maximum degree, which lies in both induced subgraphs F(a) and F(b) such thst degree of e is greater than all the vertices it dominates and delete all the edges adjacent to e.

***Step:3*** Take e is the one of the vertex in dominating set D.

***Step:4*** By Repeating the step 2 to 3 we get c is the another vertex with maximum degree which empty the vertex set of induced subgraphs of F(x).

***Step:5*** Now the resulting vertex set in the set D = {e, c} is the minimal strong dominating set of soft graph (F,A).

**V. Conclusion**

This paper proposed the concept of strong (weak) domination in soft graphs and describes certain types of strong (weak) domination in soft graph with certain related properties and proposed algorithm to find minimal strong (weak) dominating set in soft graph.

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